

P.5

MEEG331 Incompressible Fluids Laboratory

Experiment Date: ~~September 2008~~

Due Date: ~~October 2008~~

Objectives

The main objective of the experiment was to test our knowledge obtained from class of hydrostatic force on a plane surface. The lab was also a great opportunity for us to become familiar with one another as well as the lab environment itself. The first lab gave us the opportunity as a group to generate our own lab data collection sheet as well as the necessity to familiarize ourselves with the lab and the procedure.

Theoretical Background

The theory behind this experiment is to use a torroid apparatus and a simple moment equation to rationalize the hydrostatic force acting on the torroid. By measuring the difference in the height of the water in the tank as well as the weight required to balance the level, we can calculate the hydrostatic force of the water. The equation

$$P = \rho g y + P_{\text{atm}}$$

as well as the $\Sigma \text{Moment} = 0$ equation will prove quite relevant to these calculations. This experiment will demonstrate the way hydrostatic forces act on submerged surfaces, providing a great example of the topics discussed in class.

Equipment

The equipment for this experiment includes a tank of water (1), a balance arm hinged near the middle (2) with weights on one side (3), a level on the arm (4), and a torroid attached to the bottom (5). This torroid, shaped like a quarter circle of width b , is submerged with the flat face perpendicular to the water surface. It is the most important piece of equipment, as the force from hydrostatic pressure is being measured on the submerged vertical face. The equipment also included a tool for measuring water height (6), and a thermometer (7) for water temperature.

See appendix for the detailed schematic.

Procedure

Before the experiment, prepare a data table in which you will record the all measurements and masses. After emptying the tank of water, adjust the balance arm until it is level. Let the water run for a few minutes before filling the tank. Fill the tank until the water surface touches the top edge of the torroid face. Next, record water temperature using the thermometer on the back right of the tank. After reaching the desired initial water height and recording temperature, add mass to the weight basket on the left of the balance. Adjust water level by either letting water out, or adding water in small amounts until the balance is horizontal as indicated by the level. Use the device on the back left corner of the tank to measure water height. To use that device, rotate the dials until the bottom point is as close to the water as possible without breaking the surface tension. Then, record the height in your table along with the corresponding mass. Add different

masses and record the water height for 10 total trials. Make sure to cover a large range of masses for post-experiment analysis. After finishing the trials, one may use the data from the experiment to calculate the area of submerged surface, force and center of pressure on the vertical plane, and moment M_f about pivot O.

Results

See appendix for attached Excel worksheet.

Error Analysis

When deciding the sources of error it would have to come from one of the five measured values we used to find our forces. The mass was measured with weights measured in grams we assumed could vary 1 gram from what it said its mass was because the weights were old and worn. The water level height was very accurately measured with a device accurate to the .05mm. The measurements of the torroid were inaccurate measurements because we used a standard 12 inch ruler which was very difficult to line up with the different dimensions. We assumed a possible 1 mm error in those measurements. The largest percent error in our results was when the weight became very small. If the measured weight of 10 grams could vary 1 gram we would assume around a 10% error which is what we saw in our calculations of $\Delta M_w/M_w$. Clearly the measurement affecting the results the most was the weight, for better results next time we could use an accurate scale to weigh the weights before adding them to the experiment. Also using a more accurate way of measuring the dimensions of the torroid would help the accuracy of the results.

Discussion and Conclusions

After studying hydrostatic forces on a flat plane, it is fascinating to see the applications of formulas into this experiment. From this lab it can be concluded that the hydrostatic pressure acting at the center of pressure counteracts the weight in the mass pan and that the two are directly related. Either the mass or the height of the water could be forced to be the independent variable. Since it was chosen that the mass would be independent, the level of the water had to compensate for the added weight in the mass pan. As the weight grew larger, the moment about the pivot, even though the moment arm remained the same, increased. This increase demanded the level of the water to be decreased in order to create a longer moment arm for the hydrostatic pressure of the water to counteract the force of the weight. By dropping the water level, it is possible to lower the center of pressure on the face of the torroid and create a larger moment to negate the added mass. Not only did the hydrostatic pressure act on the flat plane of the torroid, but also on the circular side of the torroid; however, it did not incur more of a moment in the arm. Forces acting on a curved surface can be broken down into a horizontal and vertical force. These two forces counteract each other in the moment equation. One, the horizontal force, aides the mass pan, but the other, the vertical force, contributes to the pressure on the flat surface, thus negating each other. Also, it can be assumed that this force on the curved surface acts through the pivot point itself, also not aiding or deterring from the moment.

There were no real discrepancies between theory and practice in this experiment. The fluid pressure was hydrostatic once the balance arm was horizontal. The water level needed to be lowered, just like in theory, in order to counteract the increased weight in the mass pan. Even though it was only a minor inconvenience, the method of removing and adding water from the control volume to ensure a horizontal balance arm was tedious and difficult at some times.

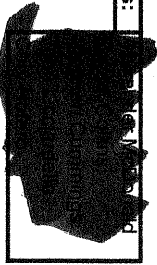
Appendix

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Lab 1:

Incompressible Fluids Laboratory

Group Members:



Measurement Number	Measured Data	
	Control Variable Height of Water	Measured Variable Mass on Scale
1	94.6 mm	210g
2	97.8 mm	200g
3	94.6 mm	190g
4	92.0 mm	180g
5	88.9 mm	150g
6	77.7 mm	130g
7	67.7 mm	100g
8	52.0 mm	60g
9	42.3 mm	40g
10	21.2 mm	10g

Calculated Data			
Force	Center of pressure	Mf	Mw

Other Measured Data

Water Temperature	23.52
R1	101.5 mm
R2	205 mm
R3	300 mm
b	77 mm

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Lab 1:

Incompressible Fluids Laboratory

Group Members:

Measurement Number	Measured Data	
	Control Variable	Measured Variable
1	Height of Water (mm) 100.0	Mass on Scale (g) 210
2	97.8	200
3	94.6	190
4	92.0	180
5	83.9	150
6	77.7	130
7	67.7	100
8	52.0	60
9	42.3	40
10	21.2	10

Calculated Data						
Area (mm ²)	Force (N)	Center of pressure From Centroid (mm)	Center of Pressure of Area From Bottom of Toroid (mm)	M _f (N·m)	M _w (N·m)	Difference between M _f and M _w
7700.0	3.768	16.67	33.33	0.639328	0.617941	0.02148737
7630.6	3.604	16.30	32.60	0.61415	0.58842	0.02573041
7284.2	3.372	15.77	31.53	0.578215	0.558999	0.01921609
7084.0	3.189	15.33	30.67	0.549632	0.529578	0.02005448
6460.3	2.652	13.98	27.97	0.464272	0.441315	0.02295661
5982.9	2.275	12.95	25.90	0.402891	0.382473	0.02041844
5212.9	1.727	11.28	22.67	0.311617	0.29421	0.0174073
4004.0	1.019	8.67	17.33	0.189177	0.176526	0.01265089
3257.1	0.674	7.05	14.10	0.127362	0.117684	0.00967808
1832.4	0.169	3.53	7.07	0.033182	0.029421	0.00376138

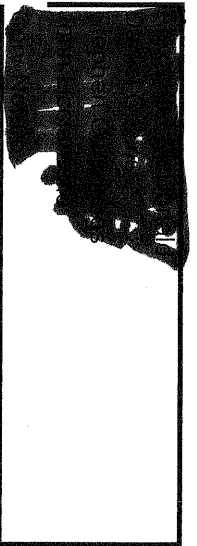


Other Measured Data	
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Water Temperature	23.5
R1	101.5
R2	203.0
R3	300.0
b	77.0

Other Data	998 kg/m ³
ρ of water	9.807 m/s ²
g	





Error delta Mf

error delta Mw

delta Mf/Mw

delta Mw/Mw

0.009136

0.003591

0.01429034

0.005812646

0.008771

0.003536

0.01428136

0.006009252

0.00825

0.003483

0.01426873

0.006229923

0.007837

0.003431

0.01425884

0.006478835

0.006607

0.003289

0.01423045

0.00745356

0.005726

0.003206

0.01421157

0.008383478

0.004421

0.003101

0.01418789

0.010540926

0.002682

0.003

0.01417859

0.016996732

0.00181

0.002968

0.01420797

0.025221243

0.000488

0.002944

0.01470257

0.10005554

Other Measured Data

Water Temperature	23.5
R1	101.5
R2	203.0
R3	300.0
b	77.0

Other Data

p of water	998	kg/m ³
g	9.807	m/s ²
delta h	0.05	mm
delta b	1	mm
delta R2	1	mm
delta R3	1	mm
delta m	1	g

7

Mf vs Mw

0.7

0.6

0.5

0.4

Mw (N*m)

0.3

0.2

0.1

0

0.1

0.2

0.3

0.4

0.5

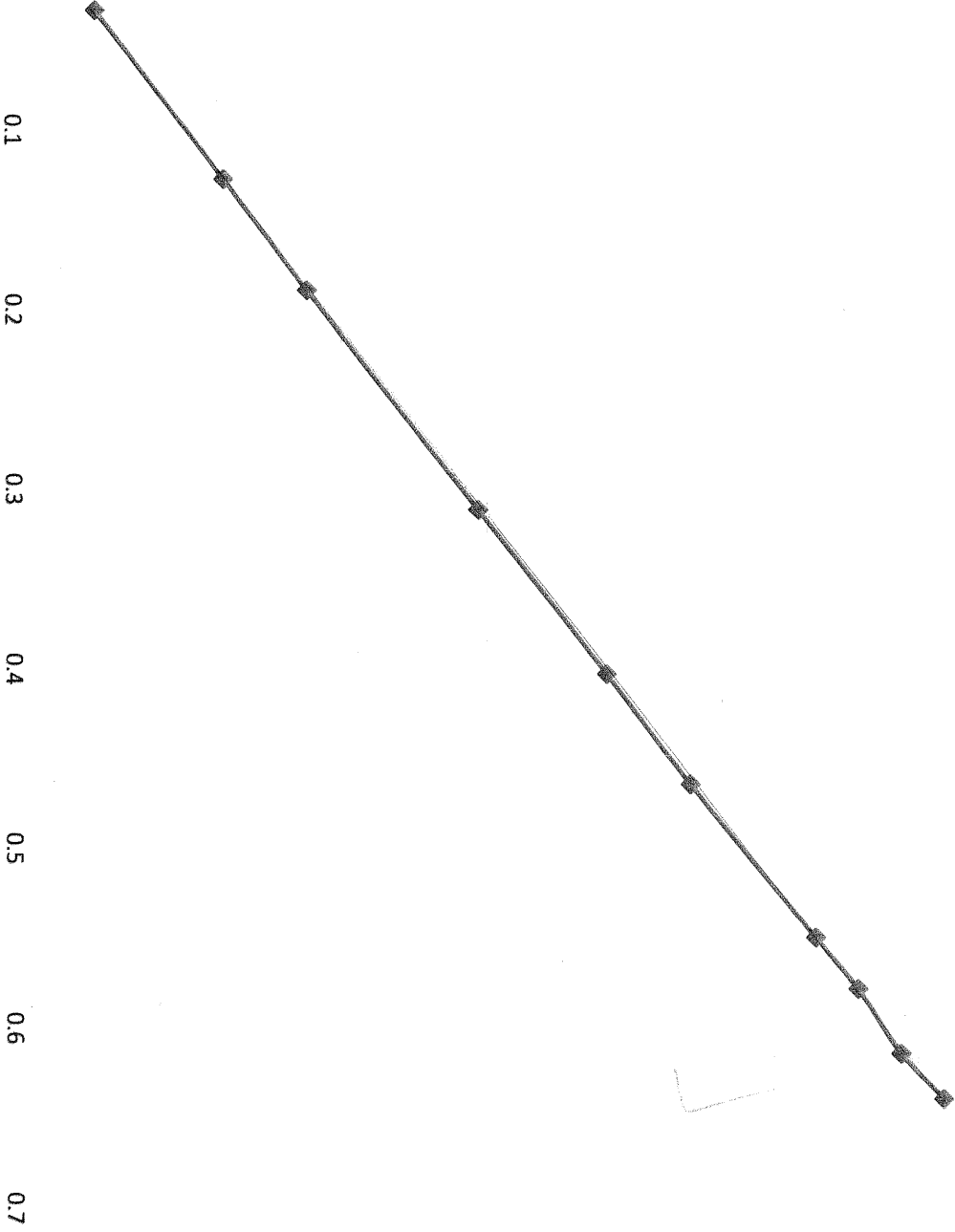
0.6

0.7

Mf (N*m)

$$Y = 0.9717X - 0.0063$$
$$R^2 = 0.9998$$

◆ Data
— Linear (Data)



Worked out Calculations

- Area of submerged surface:

b = width of Torrid Y = height of water

$$\text{Area} = b \cdot Y$$

$$A = b \cdot Y$$

Data from measurement 1

$$A = 77 \text{ mm} \cdot 100 \text{ mm} = 7700 \text{ mm}^2 \left(\frac{1 \text{ m}^2}{1000^2 \text{ mm}^2} \right) =$$

repeated on excel

- Center of pressure y_{cp} from (C.G.)

$$\sin \theta = 1$$

$$y_{cp} = - \frac{I_{xx} \sin \theta}{h_{cg} \cdot A}$$

$$A = b \cdot Y$$

$$y_{cp} = - \frac{b Y^3}{12} / (b \cdot Y) (Y/2)$$

$$h_{cg} = Y/2 \text{ (for rectangles)}$$

$$y_{cp} = - \frac{b Y^3}{12 \left(\frac{b Y^2}{2} \right)}$$

$$y_{cp} = - \frac{b Y^3}{6 b Y^2} = - \frac{Y}{6} \quad \frac{Y/2}{3}$$

Data from measurement 1

$$y_{cp} = - \frac{100 \text{ mm}}{6} = 16.67 \text{ mm}$$

repeat on excel

- Force

$$F = \rho g h_{cg} \cdot A$$

ρ for water at 23.5°C $\approx 998 \text{ kg/m}^3$

$$g = 9.807 \text{ m/s}^2$$

$$F = \rho g \cdot \frac{Y}{2} \cdot b \cdot Y$$

Data From measurement 1

$$F = 998 \cdot 9.807 \cdot \left(\frac{100 \text{ mm} (1 \text{ m})}{1000 \text{ mm}} \right) \cdot 5 \cdot (7700 \text{ mm}^2 / 1000^2 \text{ mm}^2) \text{ m}^2 = 3.768 \text{ N}$$

repeated on excel. ✓

• Moment M_f

$$M_f = F \cdot D$$

$$M_f = F \cdot (R_2 - (Y/2 - Y_{cp}))$$

Data from measurement 1.

$$M_f = 3.768 \text{ N} \left(\frac{203.0}{1000} - \left(\frac{100 \text{ mm}}{2000} \right) - 16.67/1000 \right) = .639328 \text{ N}\cdot\text{m}$$

repeated on excel

• Moment M_w

$$M_w = M \cdot g \cdot D$$

$$M_w = M \cdot g \cdot R_2$$

Data from measurement 1

$$M_w = \left(\frac{2109}{1000} \right) (9.807 \text{ m/s}^2) \cdot \left(\frac{300}{1000} \right) = .617841 \text{ N}\cdot\text{m}$$

repeated on excel

Error analysis

$$M_f = \rho g \frac{h}{2} (hb) \left(R_2 - \left(\frac{h}{2} - \frac{h}{6} \right) \right)$$

$$M_f = f(b, h, R_2) = C \left(bh^2 R_2 - \frac{bh^3}{3} \right) \quad C = \frac{\rho g}{2}$$

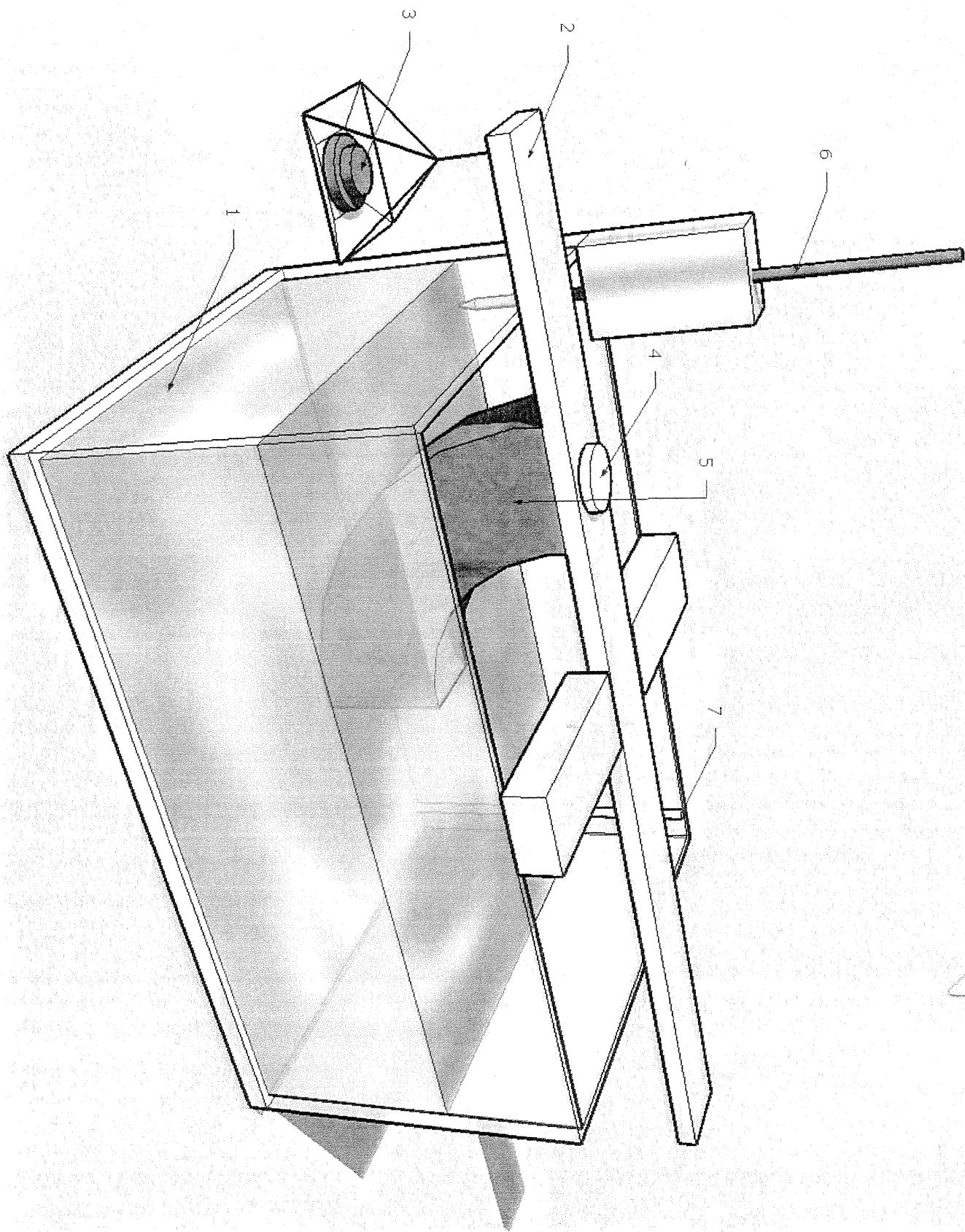
$$\Delta M_f = \sqrt{\left(\frac{\partial f}{\partial h} \Delta h \right)^2 + \left(\frac{\partial f}{\partial b} \Delta b \right)^2 + \left(\frac{\partial f}{\partial R_2} \Delta R_2 \right)^2}$$

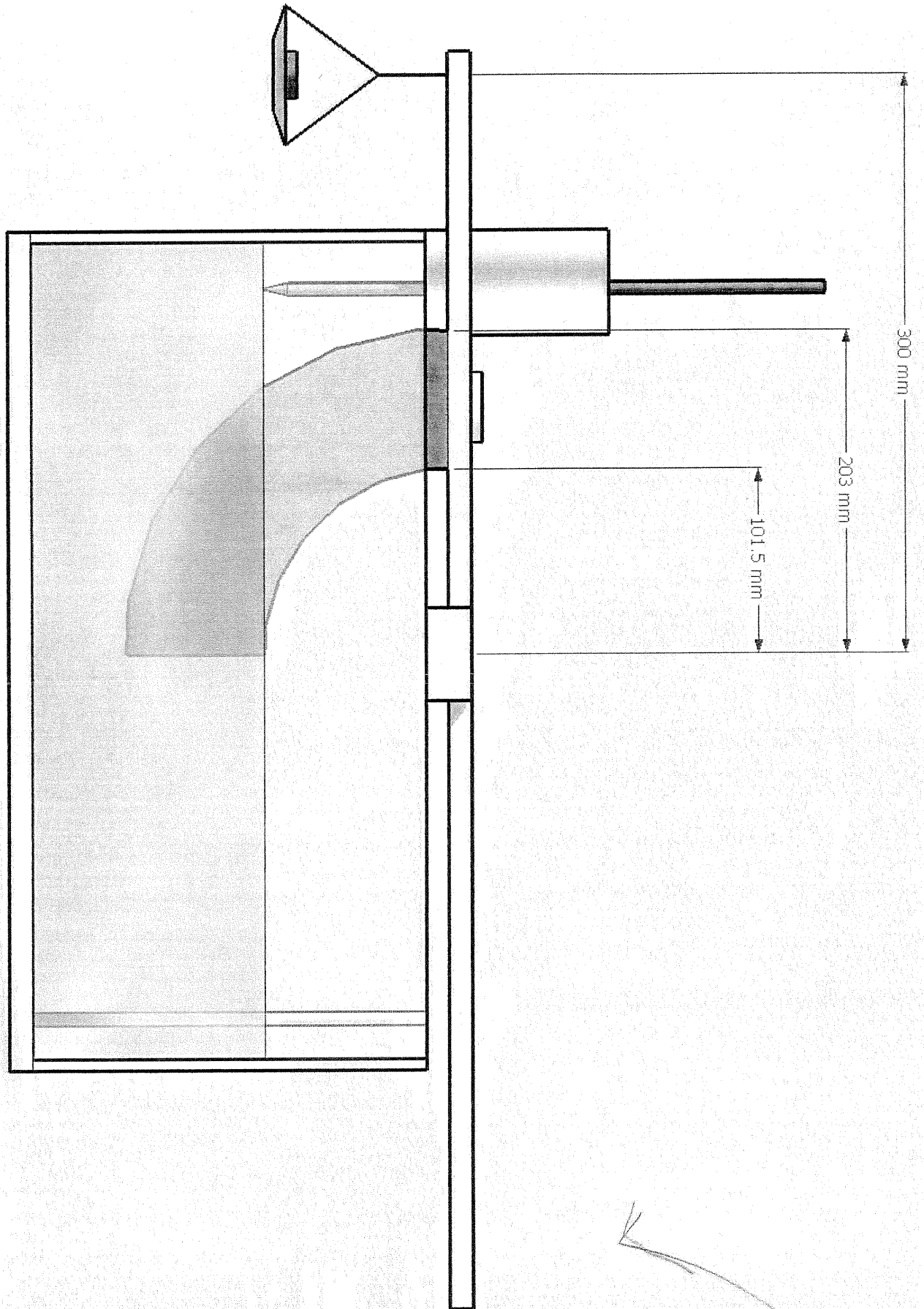
$$\Delta M_f = \sqrt{\left((2C b R_2 h - C b h^2) \Delta h \right)^2 + \left((C h^2 R_2 - \frac{C h^3}{3}) \Delta b \right)^2 + \left((C b h^2) \Delta R_2 \right)^2}$$

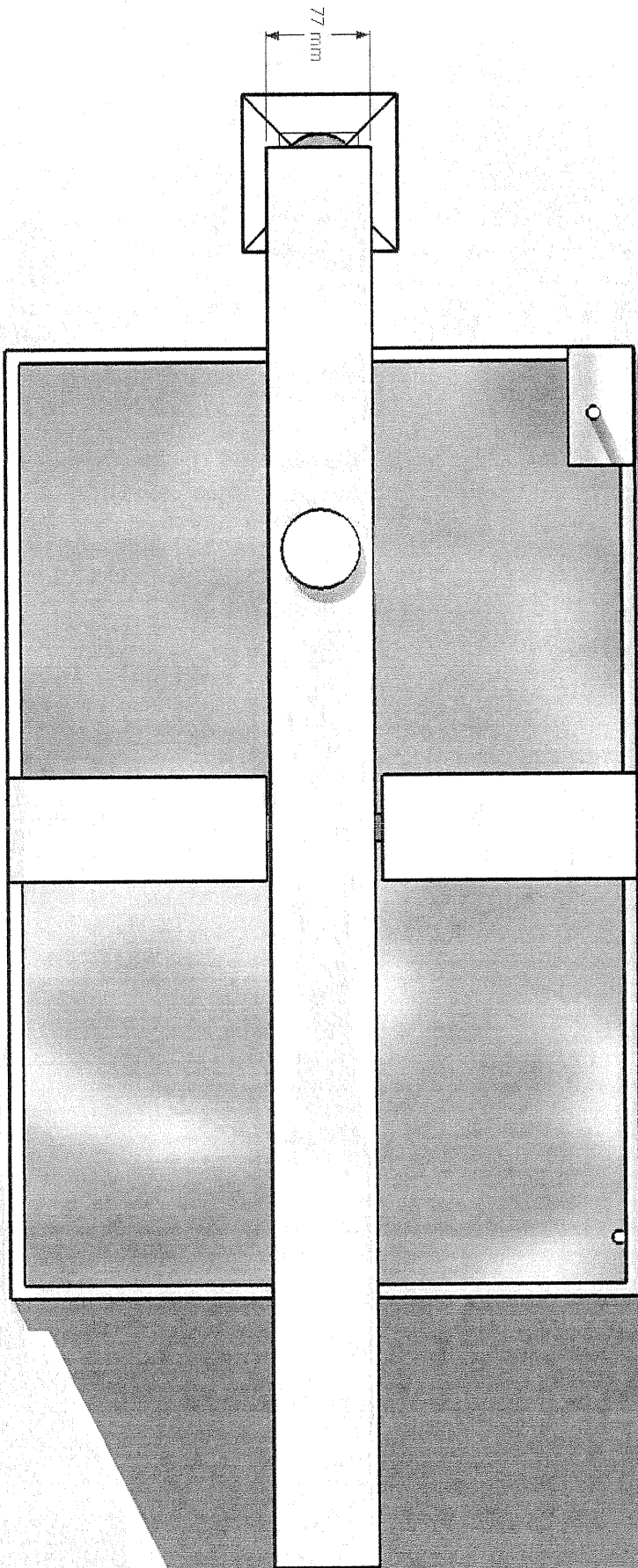
measurement 1 Data error

$$\Delta M_f = \sqrt{\left(\frac{2(9.8)(998)(.077)(.203)(.1) - (9.8)(998)(.077)(.1)^2}{2} (.0005) \right)^2 + \left(\frac{(9.8)(998)(.1)^2(203) - (9.8)(998)(.1)^3}{2 \cdot 3} (.001) \right)^2 + \left(\frac{(9.8)(998)}{2} (.077)(.1)^2 (.001) \right)^2} \cdot \frac{1}{2}$$

$$= .00136$$







✓