MEEG 215: Mechanical Response of Materials

Lab Objectives

1) Study the deformation response of carbon steel to static and dynamic loadings.
2) Determine the 95% confidence interval for elastic modulus, E, of several materials.
3) Determine the 95% confidence interval for ultimate strength, Su, of several materials.
4) Experimentally establish a model for the stiffness of an axial member.
5) Apply your model and results to the design an axial member for a truss.

Background

You have already collected a limited failure dataset in the previous lab. You should have found:

1) that results and interpretations can be skewed by measurement uncertainty and bias;
2) that the strength of wire in a single spool has a distribution of values;
3) that the break force is insensitive to length;
4) that the break force is proportional to the wire cross-sectional area

The hypothesis in the first lab was that the break force would be a function of the length and cross sectional area. To start a functional analysis, we could assume a relationship: $F_{br} = C \cdot L^\alpha \cdot A^\beta$, where $C$ is a material constant, $L$ is length and $A$ is area. Next, we would need to isolate each potential variable; $i.e.$ only one variable is changed at a time.

Figure 1. Results from Lab1, station 4. Left: Break force plotted versus wire length for carbon steel wires of 0.009in diameter. Error bars represent a 95% confidence interval for the measurement uncertainty. Right: Break force plotted versus wire cross-sectional area for lengths of carbon steel wire from 5-19”. It was found that break force is insensitive to length and varies linearly with area.
Break force is plotted versus length of 0.009” wire on the left of Figure 1. A power law fit to this data gives $F_{br} = 25.3L^{0.001}$. This tells us that break force is insensitive to length and $L^{x}$ can be replaced by $L^{0}=1$. This also makes intuitive sense since the variation in break force with length was smaller than the sample to sample variation at the 12” length condition. Now we can include variable length measurements in our analysis of area. Break force is plotted versus area on the right of Figure 1 for multiple sample lengths. A power law fit gives $F_{br} = 276000A^{0.9631}$ independent of $L$. This tells us that $\beta=1$ and the relationship is linear to a very good approximation. In every case, imperfect data will produce non-integer power fits. Non-integer power relationships rarely make physical sense, so it is up to you to judge your results appropriately. Sometimes an outlier (bad measurement) can throw off a fit and sometimes the variable states are too close (e.g. if lengths were 17.4” and 17.5” instead of 12” and 17.5”). It will be your job to judge the results based on your experiences and observations during the lab (kinks, break locations, improper caliper use, etc...). You will use this procedure to analyze your results in this lab.

Break force can now be said to follow the general function, $F_{br} = C \cdot A$, where $F_{br}$ is the break force, $C$ is a material constant and $A$ is the cross-sectional area. The constant, $C$, can be found by dividing $F_{br}$ by $A$. The quantity $F/A$ can be thought of as a force intensity; in engineering, this force intensity is defined as the normal stress, $\sigma$. When the stress exceeds $F_{br}/A$, the sample breaks; this critical value of normal stress is defined as the ultimate strength, $S_u$, which is identical to the constant, $C$. From 11 measurements of break force for carbon steel wire at station four, the average $S_u$ was 392,700 psi and the standard deviation was 5,500 psi.

### Uncertainty Analysis

An engineer must use quantitative ultimate strength data to design effective axial members. Calculating the ultimate strength, $S_u$, from 11 measurements gives $\mu = 393$ ksi and $\sigma = 6$ ksi (ksi is the customary unit and represents 1,000 psi). To understand the influence of our measurements on the results, we need to understand the measurement uncertainty.

Force and diameter are directly measured; these are called measurements. Ultimate strength and area are not directly measured; they are calculated from direct measurements and are called measurands. These individual measurement uncertainties in force and diameter are propagated in the calculations of area and ultimate strength according to the law of propagation of uncertainty. The law of propagation of uncertainty is used to calculate the combined standard uncertainty of a measurand as follows:

$$U^2_c f(x_1, x_2 ... x_n) = \sum_{i=1}^{n} S_{x_i} = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$$

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1 Note that the symbol, $\sigma$, is used in engineering for stress and in statistics for standard deviation. This can lead to confusion. You should be able to use sentence context to determine which is under discussion.
For any function, \( f \), of independent variables \( x_1, x_2, \ldots, x_n \), the square of the combined standard uncertainty of that function (the measureand) is the sum of each sensitivity factor, where the sensitivity factor of a variable \( x \) is the square of the partial derivative of \( f \) with respect to \( x \) times the square of the uncertainty of \( x \). For example, the cross-sectional area is indirectly calculated using a direct diameter measurement.

To find the uncertainty in area, we first express it in terms of each measurement as follows; 

\[
A = \frac{\pi D^2}{4} .
\]

According to the law of propagation of uncertainty,

\[
U_c^2 (A) = \left( \frac{\partial A}{\partial \pi} \right)^2 u(\pi)^2 + \left( \frac{\partial A}{\partial D} \right)^2 u(D)^2
\]

Since there is no uncertainty in the values 4 and \( \pi \), the expression reduces to

\[
U_c^2 (A) = \left( \frac{\partial A}{\partial D} \right)^2 u(D)^2 = \left( \frac{\pi D}{2} \right)^2 u(D)^2
\]

With proper use, it is reasonable to claim a caliper measurement uncertainty of 0.0003"; i.e. for a measurement of 0.009", the true diameter lies in between 0.0087" and 0.0093" with 68% confidence. For this measurement, the uncertainty in \( A \) becomes;

\[
U_c^2 (A) = 0.0002 \text{ in}^2 \cdot 9 \times 10^{-8} \text{ in}^2 \quad \therefore U_c (A) = 4 \times 10^{-6} \text{ in}
\]

This area uncertainty is a function of the diameter measurement.

The ultimate strength, expressed in terms of individual measurements, is:

\[
S_u = \frac{4F_{br}}{\pi D^2}
\]

Eq. 1

The law of propagation of uncertainty gives,

\[
U_c^2 (S_u) = \left( \frac{\partial S_u}{\partial D} \right)^2 u(D)^2 + \left( \frac{\partial S_u}{\partial F_{br}} \right)^2 u(F_{br})^2 = \left( \frac{-8F_{br}}{\pi D^3} \right)^2 u(D)^2 + \left( \frac{4}{\pi D^2} \right)^2 u(F_{br})^2
\]

Eq. 2

The force uncertainty must be determined. The load cell was calibrated with 5 kg weights having an uncertainty of 5 g. This corresponds to an uncertainty of 0.01 lb/10lb or 0.001lb/lb. For the first measurement from station 4, the break force for a 0.009" diameter wire was 25.19 lb. The diameter uncertainty is 0.0003" and the force uncertainty is 0.025lb. Equation 1 gives a nominal ultimate strength of \( S_u = 396 \text{ ksi} \). Equation 2 provides the uncertainty in this value which turns out to be 26 ksi. This says that the ultimate strength is known to lie in between 370 and 422 ksi with 68% confidence. The instrument repeatability and material variability are overpowered by measurement uncertainty. Hand calculations of measurement uncertainty are cumbersome, so it is standard practice to set up the equation in Excel for repeated calculations.
We can reduce this confidence interval by improving measurement uncertainty. Only in rare instances will each measurement contribute equally to the measurand uncertainty. We need to determine the dominant contributor by comparing the diameter and force sensitivity factors, \( \frac{S_D}{U_c^2(S_u)} \) and \( \frac{S_F}{U_c^2(S_u)} \); the first is 0.9997 and the second is 0.0003. Therefore, the diameter uncertainty accounts for 99.97% of the uncertainty in ultimate strength. This tells us that we will only benefit from improvement to our diameter uncertainty.

A measurement of the wire with micrometers gives an improved diameter measurement of 0.0088 and an uncertainty of 0.0001” (68% confidence). The ultimate strength is now \( Su = 414 \text{ ksi} \pm 10 \text{ ksi} \) with 68% confidence. Our uncertainty is significantly reduced if micrometers are used in place of the calipers.

**Mechanical Deformation**

Materials deform in response to an applied stress. The change in length of a specimen per unit length of material is the **strain**, \( \varepsilon \). The relationship between stress and strain depends on the material. Ideally, stress is proportional to strain; the constant of proportionality is a material constant, \( E \), called the **Young’s modulus or elastic modulus**. In the ideal sense, the yield strength is the stress above which the material becomes permanently deformed. The transition to permanent deformation is gradual, so a different definition must be used in practice. Yield strength is defined as the stress needed to produce a permanent strain of 0.002 or 0.2%. The basic properties of the idealized stress-strain diagram are shown below.

![Figure 3. Schematic of a material under load and a stress-strain diagram](image)
Imagine performing a tension experiment and finding the stress-strain curve shown in Figure 3. If a second identical sample were loaded to a stress lower than the yield strength and released, the loading and unloading curves would not exactly overlap. If the endpoints are the same, the behavior is elastic and nonlinear and the result is energy dissipation in the form of heat. The amount of energy dissipated per cycle per unit volume is the area bound by the loading and unloading curves. This is the same effect responsible for the heating of a rubber band during rapid load cycling. **Mechanical hysteresis** occurs because all materials require some small amount of time to deform in response to an applied load.

![Figure 4. Illustrations of cyclic hysteresis (left), strain hardening (center) and creep (right).](image)

If a sample is brought above its yield point, it will become permanently deformed. Above the yield point, the stress required to cause slip is reached at locations where slip is most favorable. In brittle materials (like glass), stress above this point results in fracture. Many metals become stronger following yield for reasons that include grain refinement reorientation, and interactions of impurities and defects (called dislocations). If a sample were loaded past the yield point and released, a permanent deformation would be observed. If the sample were re-loaded, it would not yield until it reached a load of $S_{Y2}$. This phenomenon is called **strain hardening**.

Materials that plastically deform are also susceptible to creep. Creep is deformation that occurs with time at a constant stress. If a plastic sample were loaded to a high constant stress, the sample would continue to stretch with time until it failed. Creep rates depend on the material and increase with increased stress and temperature.

**Procedure**

1) **Setup for mechanical response measurements**
   a) Verify calibration constants with standard samples and record measurements
   b) Measure and record lab temperature with the voltmeter.
   c) Create a folder for your group on the desktop.
   d) Restart the software and click on the blue ‘save data’ button to select a file path destination.
   e) Create a folder (1) in your group folder and save your first file there.
   f) Increase the save interval to 2000 seconds to avoid saving data from an improper setup
   g) Click ‘set files to start measurements’.
2) **Study cyclic loading**

   a) Section the 0.014” carbon steel wire, measure diameter and zero the load. (Hint: you want to perform experiments with the same length conditions if possible, so fixture a length you can easily repeat)
   b) Fasten in the wire grips and apply approximately 10 lb of tension to remove slack.
   c) measure the test length\(^2\)
   d) Set the safety collar.
   e) Insert LVDT into holder and lock into position when it reads between -8V and -9V (to maximize range), then zero displacement.
   f) Increase the acquisition rate to 10,000 samples per second to reduce acquisition time.
   g) Set average save interval to 0.2 seconds
   h) Slowly increase the force to 30 lb and note the relationship between force and displacement.
      Describe what is happening in your report.
   i) Unload the sample to 10 lb and reload to 30 lb. Did the force-displacement curve follow the previous path?

3) **Study yielding and creep**

   a) **The sample may be close to breaking near yield, so be careful with the load.** Increase the load to find the yield point. Estimate the yield force and reduce to 10 lb.
   b) If you have hit the yield point, there will be a shift to greater displacement for any load. Why?
   c) Increase the load to induce yield again. Did the yield strength increase?
   d) **Up to 5 point bonus:** investigate creep.

4) **Determine modulus and stiffness**

   a) The slope of the force-displacement curve is defined as the stiffness or spring-rate, k. Increase load to below the yield point. Test elastic modulus and spring-rate repeatability by quickly cycling load from 10 lb to just below the yield point. Comment on the behavior.
   b) Stop the labview software with the stop button next to run.
   c) Open your data file and insert columns for stress and strain. Insert the appropriate formula and plot stress versus strain and force versus displacement for the data of interest (the last load ramp is the easiest to find)
   d) Use a trendline or the ‘slope’ function to calculate slopes. Check your modulus value using the internet before continuing. Check with your TA if your measurement is off by more than 30%. Continue after confirming the procedure. You will save your data to a USB drive and perform the rest of the stiffness calculations after lab.

5) **Study the effects of wire diameter on spring rate**

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\(^2\) The test length is the length of the sample that is subject to stress. This is difficult to determine since some slip occurs along the friction roller. This effect introduces additional uncertainty into the length measurement.
a) Section a 0.009” carbon steel wire, start program, save to your group folder in a sample 2 folder, increase the average save interval to 2000 seconds and set files.

b) Zero load, measure diameter, mount wire taking care to repeat your first length condition, remove slack (5-7lb) and measure length.

c) Set collar, mount LVDT, zero displacement

d) Yield the wire, reduce load and conduct the stiffness test.

e) Stop test.

f) Repeat for at least 1 more wire diameter of the carbon steel at the same length

6) Study the effects of wire length on spring rate

a) Use a carbon steel wire that has been measured in a previous section

b) Raise the piston by at least two inches and repeat modulus and stiffness measurements

c) Lower the cross-bar and repeat the modulus and stiffness measurements for a 5 inch sample. Continue loading to failure.

7) Determine material effects on stiffness and ultimate strength

a) Repeat the stiffness measurements for stainless steel, bronze and copper wires. In this case, you will continue loading the sample to failure for ultimate strength calculations. Set the safety collar once reaching a load of 35 lb. Overlap the length and diameter conditions as much as possible to eliminate potential ‘cross-talk’ effects between variables. Investigate linearity, hysteresis, creep, hardening, and other phenomena as time permits. Investigate aluminum and Trilene as time permits.
Analysis

1) Plot and discuss the cyclic behavior of the carbon steel. Compare with other materials if available – 5 pts.
2) Plot and discuss the strain hardening results. Compare with other materials if available – 5 pts.
3) Determine modulus, stiffness, modulus uncertainty and stiffness uncertainty for each test. – 25 pts
4) Plot modulus and stiffness versus wire cross-sectional area for the variable diameter experiments. Determine the relationship between stiffness and area as was we did for break force and area. – 15 pts
5) Plot modulus and stiffness versus length for the variable length experiments. Determine the relationship between stiffness and length – 15 pts
6) Determine the relationship between stiffness and modulus. Provide an equation for the stiffness in terms of length, diameter and modulus – 20 pts
7) Determine the ultimate strength and uncertainty for each material (include lab 1 data) – 5 pts

Hint: You will calculate the modulus with the equation $E = \frac{\Delta \sigma}{\Delta \varepsilon} = \frac{4 \cdot \Delta F \cdot L}{\pi \cdot d^2 \cdot \Delta L}$ since the initial point will be nonzero. Therefore, the modulus uncertainty depends on force, diameter, displacement and length uncertainties; $U^2(E) = \left(\frac{\partial E}{\partial D}\right)^2 u(D)^2 + \left(\frac{\partial E}{\partial \Delta F}\right)^2 u(\Delta F)^2 + \left(\frac{\partial E}{\partial L}\right)^2 u(L)^2 + \left(\frac{\partial E}{\partial \Delta L}\right)^2 u(\Delta L)^2$. For an additional 5% bonus, determine which measurement dominates the modulus uncertainty and describe how you would improve the measurement uncertainty.

Design Problem - 10 pts

You will use your results to solve a truss design problem. A truss is a structure whose members experience only axial loads. The truss shown to the right is designed to support a load, $F=1000\text{lb}$, without breaking. The truss consists of a compression member, bc, and a tension member, ac. Pin joints b and a are 10” apart. Use your results to recommend an appropriate tension wire material and length (wire length with determine length b-c). The truss will be used outdoors, so it needs to resist...
corrosion. Your target is to minimize $\alpha$ while maximizing the probability of long-term survival using these available wires:

1) Carbon steel: $\phi = 0.08''$
2) Stainless steel: $\phi = 0.08''$
3) Bronze $\phi = 0.10''$
4) Copper: $\phi = 0.10''$
5) Aluminum $\phi = 0.125''$
6) Trilene: $\phi = 0.25''$

Neglect deformations and failure of the compression member in this exercise. After specifying your wire and length, provide the factor of safety (stress divided by mean strength) and the maximum deflection $\alpha$, for 99.9% confidence. Your solutions must be justified.

Figure 1. Truss under design consideration. The load is 1000 lb and the vertical height is 10”. Design the tension member (ac) material and length for outdoor use, better than 99.9% survival probability and minimum angular deflection, $\alpha$, under full load.