4-39. Determine the resultant moment produced by the two forces about point \( O \). Express the result as a Cartesian vector.

**Position Vector:** The position vector \( \mathbf{r}_{OA} \), Fig. a, must be determined first.
\[ \mathbf{r}_{OA} = (3 - 0)i + (3 - 0)j + (-2 - 0)k = [3i + 3j - 2k \text{ ft}] \]

**Resultant Moment:** The resultant moment of \( F_1 \) and \( F_2 \) about point \( O \) can be determined by
\[
(M_R) = \mathbf{r}_{OM} \times (F_1 + F_2)
\]
\[
= \begin{vmatrix}
  i & j & k \\
  3 & 3 & -2 \\
-20 & 10 & 30 \\
\end{vmatrix}
\]
\[
= (200i - 180j + 30k) \text{ lb \cdot ft}
\]

Or we can apply the principle of moments which gives
\[
(M_R) = \mathbf{r}_{OM} \times (F_1 + F_2)
\]
\[
= \begin{vmatrix}
  i & j & k \\
  3 & 3 & -2 \\
-30 & -20 & 80 \\
\end{vmatrix}
\]
\[
= (200i - 180j + 30k) \text{ lb \cdot ft}
\]

4-43. Determine the moment produced by each force about point \( O \) located on the drill bit. Express the results as Cartesian vectors.

**Position Vector:** The position vectors \( \mathbf{r}_{MA} \) and \( \mathbf{r}_{OB} \), Fig. a, must be determined first.
\[ \mathbf{r}_{MA} = (0.15 - 0)i + (0.3 - 0)j + (0 - 0)k = [0.15i + 0.3j \text{ m}] \]
\[ \mathbf{r}_{OB} = (0 - 0)i + (0.6 - 0)j + (-0.15 - 0)k = [0.6j - 0.15k \text{ m}] \]

**Vector Cross Product:** The moment of \( F_A \) about point \( O \) is
\[
(M_R) = \mathbf{r}_{MA} \times F_A
\]
\[
= \begin{vmatrix}
  i & j & k \\
  0.15 & 0.3 & 0 \\
-40 & -100 & -60 \\
\end{vmatrix}
\]
\[
= [-18i + 9j - 3k] \text{ N \cdot m}
\]

The moment of \( F_B \) about point \( O \) is
\[
(M_R) = \mathbf{r}_{MB} \times F_B
\]
\[
= \begin{vmatrix}
  i & j & k \\
  0 & 0.6 & -0.15 \\
-50 & -120 & 60 \\
\end{vmatrix}
\]
\[
= [18i + 7.5j + 30k] \text{ N \cdot m}
\]

*******************************************************************************
4-54. Determine the magnitude of the moments of the force \( F \) about the \( x, y, \) and \( z \) axes. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.

**a) Vector Analysis**

**Position Vector:**

\[
\mathbf{r}_{AB} = (4-0)i + (3-0)j + (-2-0)k = 4i + 3j - 2k \text{ ft}
\]

**Moment of Force \( F \) About \( x, y, \) and \( z \) Axes:** The unit vectors along \( x, y, \) and \( z \) axes are \( i, j, \) and \( k \) respectively. Applying Eq. 4-14, we have

\[
M_y = \mathbf{r}_{AB} \times F
\]

\[
= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 12 \\ \end{bmatrix} = 1[4(-3) - (12)(-2)] - 0 + 0 = 15.0 \text{ lb-ft} \quad \text{Ans}
\]

\[
M_z = \mathbf{r}_{AB} \times F
\]

\[
= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 12 \\ \end{bmatrix} = 0 - 1[4(3) - (12)(-2)] + 0 = 4.00 \text{ lb-ft} \quad \text{Ans}
\]

**Scalar Analysis**

\[
M_x = \sum M_x \quad M_y = \sum M_y \quad M_z = \sum M_z
\]

\[
M_x = 12(2) - 3(3) = 15.0 \text{ lb-ft} \quad \text{Ans}
\]

\[
M_y = \sum M_y \quad M_z = -4(2) + 3(4) = 4.00 \text{ lb-ft} \quad \text{Ans}
\]

\[
M_x = \sum M_x \quad M_y = -4(3) + 12(4) = 36.0 \text{ lb-ft} \quad \text{Ans}
\]

4-75. If \( F = 200 \text{ lb} \), determine the resultant couple moment.

**a)** By resolving the 150-lb and 200-lb couples into their \( x \) and \( y \) components, Fig. \( a \), the couple moments \( (M_C)_1 \) and \( (M_C)_2 \) produced by the 150-lb and 200-lb couples, respectively, are given by

\[
(M_C)_1 = -150 \cos 30^\circ(4) - 150 \sin 30^\circ(4) = -819.62 \text{ lb-ft}
\]

\[
(M_C)_2 = 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(2) = 560 \text{ lb-ft}
\]

Thus, the resultant couple moment can be determined from

\[
\sum (M_C)_R = (M_C)_1 + (M_C)_2
\]

\[
= -819.62 + 560 = -259.62 \text{ lb-ft} = 260 \text{ lb-ft (clockwise)} \quad \text{Ans.}
\]

**b)** By resolving the 150-lb and 200-lb couples into their \( x \) and \( y \) components, Fig. \( a \), and summing the moments of these force components algebraically about point \( A \),

\[
\sum (M_C)_R = \sum M_a \quad (M_C)_R = -150 \sin 30^\circ(4) - 150 \cos 30^\circ(6) + 200\left(\frac{4}{5}\right)(2) + 200\left(\frac{3}{5}\right)(6)
\]

\[
= -259.62 \text{ lb-ft} = 260 \text{ lb-ft (clockwise)} \quad \text{Ans.}
\]
4-97. In order to turn over the frame, a couple moment is applied as shown. If the component of this couple moment along the x axis is \( M_x = (-20) \) N·m, determine the magnitude \( F \) of the couple forces.

\[
M_c = F (1.5)
\]

Thus

\[
20 = F (1.5) \cos 30^\circ
\]

\[
F = 15.4 \text{ N} \quad \text{Ans}
\]

4-115. Handle forces \( F_1 \) and \( F_2 \) are applied to the electric drill. Replace this force system by an equivalent resultant force and couple moment acting at point \( O \). Express the results in Cartesinan vector form.

\[
F_1 = \Sigma F: \quad F_1 = 6i - 3j - 10k + 2j - 4k
\]

\[
= [6i - 1j - 14k] \text{ N} \quad \text{Ans}
\]

\[
M_{xO} = \Sigma M_O:
\]

\[
M_{xO} = \begin{vmatrix}
1 & j & k \\
6 & -3 & -10
\end{vmatrix} + \begin{vmatrix}
1 & j & k \\
0 & -0.25 & 0.3
\end{vmatrix}
\]

\[
= 0.94i + 3.30j - 0.450k + 0.41
\]

\[
= (1.30i + 3.30j - 0.450k) \text{ N·m} \quad \text{Ans}
\]

Note that \( F_{xO} = -14 \) N pushes the drill bit down into the stock.

\( (M_{xO})_x = 1.30 \) N·m and \( (M_{xO})_y = 3.30 \) N·m cause the drill bit to bend.

\( (M_{xO})_x = -0.450 \) N·m causes the drill case and the spinning drill bit to rotate about the \( z \)-axis.

\[
F_2 = [2j - 4k] \text{ N}
\]

\[
F_1 = [3i - 3j - 10k] \text{ N}
\]
Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point B.

**Equivalent Resultant Force:** Forces $F_1$ and $F_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$
\Sigma (F_R)_x = \Sigma F_x; \quad (F_R)_x = 250 \left( \frac{3}{5} \right) - 500 \cos 30^\circ \cdot 300 = -533.01 \text{ N} = 333.01 \text{ N}
$$

$$
\Sigma (F_R)_y = \Sigma F_y; \quad (F_R)_y = 500 \sin 30^\circ \cdot 250 \left( \frac{3}{5} \right) = 100 \text{ N}
$$

The magnitude of the resultant force $F_R$ is given by

$$
F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{533.01^2 + 100^2} = 542.31 \text{ N} = 542 \text{ N}
$$

The angle $\theta$ of $F_R$ is

$$
\theta = \tan^{-1} \left( \frac{(F_R)_y}{(F_R)_x} \right) = \tan^{-1} \left( \frac{100}{533.01} \right) = 10.63^\circ = 10.6^\circ
$$

**Location of the Resultant Force:** Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A,

$$
\Sigma M_R = \Sigma M_B; \quad -533.01(d) = -500 \cos 30^\circ (0) - 500 \sin 30^\circ (0.2) - 250 \left( \frac{3}{5} \right) (0.5) - 300 (2)
$$

$$
d = 2.17 \text{ m}
$$

***********************************************
Three parallel bolting forces act on the circular plate. Determine the resultant force, and specify its location $(x, z)$ on the plate. $F_A = 200$ lb, $F_B = 100$ lb, and $F_C = 400$ lb.

**Equivalent Force:**

$$F_y = \Sigma F_y; \quad -F_y = -400 - 200 - 100$$

$$F_y = 700 \text{ lb} \quad \text{Ans}$$

**Location of Resultant Force:**

$$M_x = \Sigma M_x; \quad 700(z) = 400(1.5) - 200(1.5 \sin 45^\circ)$$

$$-100(1.5 \sin 30^\circ)$$

$$z = 0.447 \text{ ft} \quad \text{Ans}$$

$$M_y = \Sigma M_y; \quad -700(x) = 400(1.5 \cos 45^\circ) - 200(1.5 \cos 30^\circ)$$

$$-100(1.5 \cos 30^\circ)$$

$$x = -0.117 \text{ ft} \quad \text{Ans}$$